Optimal Order Routing in Fragmented HF Markets

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Joint work with Costis Maglaras and Ciamac C. Moallemi.
Modern U.S. Equity Markets

- Electronic
- Decentralized
  NYSE, NASDAQ, ARCA, BATS, Direct Edge, ...
- Exchanges (≈ 70%)
  limit order books (LOBs)
- Alternative venues (≈ 30%)
  ECNs, dark pools, internalization, OTC market makers, etc.
- Participants increasingly automated
  investors: “algorithmic trading”
  market makers: “high-frequency trading”
Motivation / Contributions

How to analyze fragmented LOBs?

- each a multi-class queueing system with complex dynamics
- in addition, agents exert control via “smart order routing”
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We propose a tractable model to analyze decentralized LOBs, incorporates:

- limit order routing capability
- market order routing capability
- time-money tradeoff heterogeneity
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In this framework, we:
  • characterize through a fluid model the equilibrium
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In this framework, we:

- characterize through a fluid model the equilibrium
- establish that routing decisions simplify dynamics: state space collapse
- provide strong empirical findings supporting the model
Related Literature

- Market microstructure:
  Kyle; Glosten-Milgrom; Glosten; ...

- Empirical analysis of limit order books:
  Bouchaud et al.; Hollifield et al.; Smith et al.; ...

- Dynamic models and optimal execution in LOB:
  Obizhaeva & Wang; Cont, Stoikov, Talreja; Rosu; Alfonsi et al.; Foucault et al.; Parlour; Stoikov, Avellaneda, Reed; Cont & de Larrard; Predoiu et al.; Guo & de Larrard ...

- Transaction cost modeling, adverse selection, …: Madhavan; Dufour & Engle; Holthausen et al.; Huberman & Stanzl; Almgren et al.; Gatheral; Sofianos; ...

- Make/take fees, liquidity cycles, etc.: Foucault et al.; Malinova & Park

- Stochastic models of multi-class queueing networks
The Limit Order Book (LOB)

- Buy limit order arrivals
- Sell limit order arrivals
- Market sell orders
- Market buy orders
- Cancellations
- Cancellations
The Limit Order Book (LOB)

- **BID**
  - buy limit order arrivals
  - cancellations

- **ASK**
  - cancellations
  - market sell orders
  - sell limit order arrivals

- **Market**
  - market buy orders

Price range from lowest to highest.

5
Multiple Limit Order Books

Price levels are coupled through protection mechanisms (Reg NMS).

We consider the evolution of:
- one side of the market
- the ‘top-of-the-book’, i.e., national best bid queues across all exchanges

national best bid/ask (NBBO)
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Time Scales

Three relevant time scales:

- **Events**: order / trade / cancellation interarrival times
  \(\sim\) ms – sec

- **Delays**: waiting times at different exchanges
  \(\sim\) sec – min

- **Rates**: time-of-day variation of flow characteristics
  \(\sim\) min – hrs
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Order placement decisions depend on queueing delays in LOBs (our focus)
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Order placement decisions depend on queueing delays in LOBs (our focus)

- assume constant arrival rates of limit orders and trades
- order sizes are small relative to overall flow over relevant time scale
- overall limit order and trade volumes are high
**One-sided Multi LOB Fluid Model**

**Fluid model:** Continuous & deterministic arrivals of infinitesimal traders
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Fluid model:

- Continuous & deterministic arrivals of infinitesimal traders
- Market order
- Optimized limit order flow
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One-sided Multi LOB Fluid Model

**Fluid model:** Continuous & deterministic arrivals of infinitesimal traders

\[ \lambda_1 \rightarrow \lambda_2 \rightarrow \ldots \rightarrow \lambda_n \rightarrow \Lambda \rightarrow \text{market order} \rightarrow \text{exchange 1} \rightarrow \text{exchange 2} \rightarrow \ldots \rightarrow \text{exchange } N \rightarrow \mu_1 \rightarrow \mu_2 \rightarrow \ldots \rightarrow \mu_N \rightarrow \text{market order flow} \]

- **Dedicated limit order flow**
- **Optimized limit order flow**
The Limit Order Placement Decision

Factors affecting limit order placement:
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  \[ \Rightarrow \gamma \sim 10^1 \text{ to } 10^4 \text{ seconds per } \$0.01 \]
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- allows choice amongst Pareto efficient $(\tilde{r}_i, \text{ED}_i)$ pairs
- Implicit option for a market order: $r_0 \ll 0, \text{AS}_0 = \text{ED}_0 = 0$
The Market Order Routing Decision

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$$\arg\min_i \{ r_i : Q_i > 0, \ i = 1, \ldots, N \}$$
Market orders execute immediately, no queueing or adverse selection

Market orders incur fees ($\approx r_i$)

Natural criterion is to route order according to

$$\arg\min \{ r_i : Q_i > 0, \; i = 1, \ldots, N \}$$

Routing decision differs from “fee minimization” due to

- Order sizes are not infinitesimal; may have to be split across exchanges
- Latency to exchange introduces notion of $P(\text{fill})$ when $Q_i$ are small
- Not all flow is “optimized”, or has other economic considerations
- Traders avoid “clearing” queues to avoid increased price slippage
Attraction Model: Bounded rationality and model intricacies motivate fitting a probabilistic model of the form

\[ \mu_i(Q) \triangleq \mu \frac{f_i(Q_i)}{\sum_j f_j(Q_j)} \]

- \( f_i(\cdot) \) captures "attraction" of exchange \( i \):
  \[ \uparrow \text{ in } Q_i \text{ and } \downarrow \text{ in } r_i \]

- Remainder of this talk uses:
  \[ f_i(Q_i) \triangleq \beta_i Q_i \]
  (we imagine \( \beta_i \sim 1/r_i \))
Given \((\Lambda, \mu)\) study mean-field approximation of coupled LOBs
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- **Dynamics:** Coupled ODEs describe \(\dot{Q}(t)\) dynamics
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• Given $(\Lambda, \mu)$ study mean-field approximation of coupled LOBs

• **Dynamics**: Coupled ODEs describe $\dot{Q}(t)$ dynamics

• **Convergence**: $Q(t) \to Q^*$ as $t \to \infty$

• **State space collapse**: $Q^* = g(W^*)$, where
  - 1-dimensional “aggregate market depth” process
    \[ W^* \triangleq \sum_{i=1}^{N} \beta_i Q_i^* \]
  - $g : \mathbb{R}_+ \to \mathbb{R}_+^N$ maps the aggregate depth into queue lengths
  - $g$ balances rebates/fees with delays; depends crucially on delay preference $\gamma$
\[ \pi_i(\gamma) = \text{fraction of type } \gamma \text{ investors who send orders to exchange } i \]
Fluid Model Equilibrium

\[ \pi_i(\gamma) = \text{fraction of type } \gamma \text{ investors who send orders to exchange } i \]

**Definition.** An equilibrium \((\pi^*, Q^*)\) must satisfy

(i) **Individual rationality:** for all \(\gamma\),

\[
\pi^*(\gamma) \text{ optimizes } \max_{\pi} \pi(\gamma) \pi_0(\gamma) + \sum_{i=1}^{N} \pi_i(\gamma) (\gamma_i - Q^*_i \mu_i(Q^*))
\]

subject to
\[ \pi(\gamma) \geq 0, \quad \sum_{i=1}^{N} \pi_i(\gamma) = 1. \]

(ii) **Flow balance:** for all \(1 \leq i \leq N\),

\[
\lambda_i + \Lambda \int_{0}^{\infty} \pi^*_i(\gamma) dF(\gamma) = \mu_i(Q^*)
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Workload

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- $ED_i = Q_i/\mu_i = \left( \sum_j \beta_j Q_j \right) / (\mu \beta_i) = W / (\mu \beta_i)$
Workload

- $W \triangleq \sum_{i=1}^{N} \beta_i Q_i$ is the **workload**, a measurement of aggregate available liquidity
- $W \neq$ total market depth, also accounts for time
- $ED_i = Q_i/\mu_i = (\sum_j \beta_j Q_j)/(\mu \beta_i) = W/(\mu \beta_i)$
- Workload is a sufficient statistic to determine delays
Fluid Model Equilibrium

Theorem. \((\pi^*, W^*)\) satisfy

\[
\begin{align*}
&\text{Individual rationality:} \\
&\quad \text{for all } \gamma, \pi^*(\gamma) \text{ optimizes} \\
&\quad \max \pi(\gamma) \\
&\quad \int_{\gamma=0}^{\gamma=\infty} (\pi_0(\gamma, \gamma) \gamma + \sum_{i=1}^{N} \pi_i(\gamma) (\gamma - W^* \mu \beta_i) \{\text{d}F(\gamma)}) \\
&\quad \text{subject to } \pi(\gamma) \geq 0, \sum_{i=1}^{N} \pi_i(\gamma) = 1.
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\[
\begin{align*}
&\text{Systemic flow balance:} \\
&\quad \sum_{i=1}^{N} (\lambda_i + \Lambda \int_{\gamma=0}^{\gamma=\infty} \pi^*_i(\gamma) \{\text{d}F(\gamma)}) = \mu \\
&\quad \text{if and only if } (\pi^*, W^*) \text{ is an equilibrium, where } W^* \mu \beta_i \\
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\text{maximize} \quad & \int_0^\infty \left( \pi_0(\gamma) \gamma \tilde{r}_0 + \sum_{i=1}^N \pi_i(\gamma) \left( \gamma \tilde{r}_i - \frac{W^*}{\mu \beta_i} \right) \right) dF(\gamma) \\
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Q_i^* \triangleq \left( \lambda_i + \Lambda \int_0^\infty \pi_i^*(\gamma) dF(\gamma) \right) \frac{W^*}{\mu_i\beta_i}
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$$\sum_{i=1}^{N} \lambda_i + \Lambda(1 - F(\gamma_0)) = \mu \implies \gamma_0 = F^{-1} \left(1 - \frac{\mu - \sum_{i=1}^{N} \lambda_i}{\Lambda}\right)$$
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**Incentive compatibility:**

$$\max_{i \neq 0} \gamma(\tilde{r}_i - \tilde{r}_0) - \frac{W^*}{\mu \beta_i} \leq 0 \quad \text{for all} \quad \gamma \leq \gamma_0$$
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$$\max_{i \neq 0} \gamma_0(\tilde{r}_i - \tilde{r}_0) - \frac{W^*}{\mu \beta_i} = 0$$

**Theorem.** Under mild conditions, $W^*$ is the equilibrium workload if and only if

$$W^* = \gamma_0 \mu \max_{i \neq 0} \beta_i (\tilde{r}_i - \tilde{r}_0)$$
Empirical Results

- NYSE TAQ data, millisecond timestamps
- stocks: Microsoft (MSFT), Oracle (ORCL); 9/1/2010–9/30/2010

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- Market order routing estimation

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\mu_i = \mu \frac{\beta_i Q_i}{\sum_{i'} \beta_{i'} Q_{i'}}
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<td>(\beta_{i, \text{ORCL}})</td>
<td>1</td>
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<td>1.61</td>
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‡normalized so that \(\beta_{\text{NASDAQ}} = 1\)
Adverse Selection Estimation

- AS estimated as average difference between NBBO mid-price at time of a trade versus NBBO mid-price 1 minute later

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<td>ORCL</td>
<td>ask</td>
</tr>
<tr>
<td></td>
<td>bid</td>
</tr>
<tr>
<td></td>
<td>all</td>
</tr>
</tbody>
</table>
Under our model,

\[ \mathbf{ED}_{i,t} = \frac{Q_{i,t}}{\mu_{i,t}} = \frac{W_t}{\mu_t} \cdot \frac{1}{\beta_i} \]

Therefore, the vector of expected delays

\[ \vec{\mathbf{ED}}_t \triangleq \left( \frac{Q_{1,t}}{\mu_{1,t}}, \ldots, \frac{Q_{N,t}}{\mu_{N,t}} \right) \]

should have a low effective dimension.
Under our model,

\[
\text{ED}_{i,t} = \frac{Q_{i,t}}{\mu_{i,t}} = \frac{W_t}{\mu_t} \cdot \frac{1}{\beta_i}
\]

Therefore, the vector of expected delays

\[
\vec{\text{ED}}_t \triangleq \left( \frac{Q_{1,t}}{\mu_{1,t}}, \ldots, \frac{Q_{N,t}}{\mu_{N,t}} \right)
\]

should have a low effective dimension.

<table>
<thead>
<tr>
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<th>1\textsuperscript{st} PCA factor explains</th>
<th>1\textsuperscript{st} &amp; 2\textsuperscript{nd} PCA factor explain</th>
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<tbody>
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</tr>
<tr>
<td>ORCL</td>
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<td>92%</td>
</tr>
</tbody>
</table>
Under our model,

\[ ED_{i,t} = \frac{Q_{i,t}}{\mu_{i,t}} = \frac{W_t}{\mu_t} \cdot \frac{1}{\beta_i} \]

Therefore,

\[ ED_{i,t} = \frac{1}{\beta_i} \cdot ED_{NASDAQ,t} \]
Under our model,

\[ ED_{i,t} = \frac{Q_{i,t}}{\mu_{i,t}} = \frac{W_{t}}{\mu_{t}} \cdot \frac{1}{\beta_{i}} \]

Therefore,

\[ ED_{i,t} = \frac{1}{\beta_{i}} \cdot ED_{\text{NASDAQ},t} \]

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<tr>
<th>exchange</th>
<th>slope</th>
<th>( R^2 )</th>
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<tr>
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</tbody>
</table>
State Space Collapse II

(expected delays in sec)
Under our model,

$$\hat{E}D_t = \frac{W_t}{\mu_t} \cdot \left( \frac{1}{\beta_1}, \ldots, \frac{1}{\beta_N} \right)$$

How much of the variability of ED is explained by $\hat{E}D$?

$$\% \text{ explained} = 1 - \frac{\text{Var} (\|ED - \hat{ED}\|)}{\text{Var} (\|ED\|)}$$
Under our model,

$$\hat{ED}_t = \frac{W_t}{\mu_t} \cdot \left( \frac{1}{\beta_1}, \ldots, \frac{1}{\beta_N} \right)$$

How much of the variability of ED is explained by $\hat{ED}$?

$$\%\text{ explained} = 1 - \frac{\text{Var} \left( \|ED - \hat{ED}\| \right)}{\text{Var} \left( \|ED\| \right)}$$

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<tr>
<td>ORCL</td>
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<table>
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<th>Price Average</th>
<th>Volatility Average</th>
<th>Average Daily Volume</th>
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<td></td>
<td></td>
<td>Low ($), High ($), Bid-Ask Spread ($)</td>
<td>Daily Volatility (%)</td>
<td>(shares, $10^6)</td>
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<td>AA</td>
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<tr>
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<td>BA</td>
<td>57.53, 67.73</td>
<td>0.017</td>
<td>1.8%, 5.9</td>
</tr>
<tr>
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<td>BAC</td>
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<td>1.7%, 64.5</td>
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<td>CVX</td>
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<td>1.7%, 11.1</td>
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<td>DD</td>
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<td>1.6%, 13.3</td>
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<td>HD</td>
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<td>0.010</td>
<td>1.6%, 13.4</td>
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<td>HPQ</td>
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<td>2.2%, 32.5</td>
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<tr>
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<td>0.060</td>
<td>1.5%, 6.6</td>
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<tr>
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<td>INTC</td>
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<td>0.010</td>
<td>1.5%, 63.6</td>
</tr>
<tr>
<td>Johnson &amp; Johnson</td>
<td>JNJ</td>
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<td>0.011</td>
<td>1.2%, 12.6</td>
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<tr>
<td>JPMorgan</td>
<td>JPM</td>
<td>28.53, 37.82</td>
<td>0.010</td>
<td>2.2%, 49.1</td>
</tr>
<tr>
<td>Kraft</td>
<td>KFT</td>
<td>32.70, 35.52</td>
<td>0.010</td>
<td>1.1%, 10.9</td>
</tr>
<tr>
<td>Coca-Cola</td>
<td>KO</td>
<td>66.62, 71.77</td>
<td>0.011</td>
<td>1.1%, 12.3</td>
</tr>
<tr>
<td>McDonalds</td>
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<td>1.2%, 7.9</td>
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<td>MMM</td>
<td>71.71, 83.95</td>
<td>0.018</td>
<td>1.6%, 5.5</td>
</tr>
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<td>Merck</td>
<td>MRK</td>
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<td>1.3%, 17.6</td>
</tr>
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<td>0.010</td>
<td>1.5%, 61.0</td>
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<td>Pfizer</td>
<td>PFE</td>
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<td>0.010</td>
<td>1.5%, 47.7</td>
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<td>Procter &amp; Gamble</td>
<td>PG</td>
<td>60.30, 64.70</td>
<td>0.011</td>
<td>1.0%, 11.2</td>
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<td>AT&amp;T</td>
<td>T</td>
<td>27.29, 29.18</td>
<td>0.010</td>
<td>1.2%, 37.6</td>
</tr>
<tr>
<td>Travelers</td>
<td>TRV</td>
<td>46.64, 51.54</td>
<td>0.013</td>
<td>1.6%, 4.8</td>
</tr>
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<td>United Tech</td>
<td>UTX</td>
<td>67.32, 77.58</td>
<td>0.018</td>
<td>1.7%, 6.2</td>
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<td>Verizon</td>
<td>VZ</td>
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<td>1.2%, 18.4</td>
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<td>49.94, 53.55</td>
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<td>1.1%, 13.1</td>
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<tr>
<td>Exxon Mobil</td>
<td>XOM</td>
<td>67.93, 74.98</td>
<td>0.011</td>
<td>1.6%, 26.2</td>
</tr>
</tbody>
</table>

Table 1: Descriptive statistics for the 30 stocks over the 21 trading days of September 2011. All statistics except the average bid-ask spread were retrieved from Yahoo Finance; the average bid-ask spread is a time average computed from our TAQ data set. The daily volatility is computed from closing prices over the period in question.
(b) Average queue length (number of shares at the NBBO) across stocks and exchanges.
Figure 2: Averages of hourly estimates of the expected delays and queue lengths for the Dow 30 stocks on the 6 exchanges during September 2011. Results are averaged over the bid and ask sides of the market for each stock.

(a) Average expected delay across stocks and exchanges.
<table>
<thead>
<tr>
<th></th>
<th>% of Variance Explained</th>
<th></th>
<th>% of Variance Explained</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One Factor</td>
<td>Two Factors</td>
<td>One Factor</td>
</tr>
<tr>
<td>Alcoa</td>
<td>80%</td>
<td>88%</td>
<td>JPMorgan</td>
</tr>
<tr>
<td>American Express</td>
<td>78%</td>
<td>88%</td>
<td>Kraft</td>
</tr>
<tr>
<td>Boeing</td>
<td>81%</td>
<td>87%</td>
<td>Coca-Cola</td>
</tr>
<tr>
<td>Bank of America</td>
<td>85%</td>
<td>93%</td>
<td>McDonalds</td>
</tr>
<tr>
<td>Caterpillar</td>
<td>71%</td>
<td>83%</td>
<td>3M</td>
</tr>
<tr>
<td>Cisco</td>
<td>88%</td>
<td>93%</td>
<td>Merck</td>
</tr>
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<td>87%</td>
<td>Microsoft</td>
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<tr>
<td>DuPont</td>
<td>86%</td>
<td>92%</td>
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<td>Disney</td>
<td>87%</td>
<td>91%</td>
<td>Procter &amp; Gamble</td>
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<td>General Electric</td>
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<td>94%</td>
<td>AT&amp;T</td>
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<td>Home Depot</td>
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<td>94%</td>
<td>Travelers</td>
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<td>73%</td>
<td>84%</td>
<td>Verizon</td>
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<tr>
<td>Intel</td>
<td>89%</td>
<td>93%</td>
<td>Wal-Mart</td>
</tr>
<tr>
<td>Johnson &amp; Johnson</td>
<td>87%</td>
<td>91%</td>
<td>Exxon Mobil</td>
</tr>
</tbody>
</table>

**Table 4:** Results of PCA: how much variance in the data can the first two principle components explain.
## Table 5: Pairwise delay regressions – Sept 2011

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<tr>
<th>Security</th>
<th>NASDAQ Slope</th>
<th>NASDAQ $R^2$</th>
<th>BATS Slope</th>
<th>BATS $R^2$</th>
<th>EDGX Slope</th>
<th>EDGX $R^2$</th>
<th>NYSE Slope</th>
<th>NYSE $R^2$</th>
<th>EDGA Slope</th>
<th>EDGA $R^2$</th>
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</thead>
<tbody>
<tr>
<td>Alcoa</td>
<td>0.85</td>
<td>0.83</td>
<td>0.95</td>
<td>0.93</td>
<td>0.90</td>
<td>0.76</td>
<td>0.72</td>
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<td>0.68</td>
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<td>0.92</td>
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<tr>
<td>Procter &amp; Gamble</td>
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<td>0.93</td>
<td>0.91</td>
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<td>0.94</td>
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<td>0.84</td>
<td>0.79</td>
<td>0.92</td>
<td>0.61</td>
<td>0.89</td>
</tr>
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</table>

Table 5: Linear regressions of the expected delays of each security on a particular exchange, versus that of the benchmark exchange (ARCA).
Table 6: The measure of performance $R^2_*$, which given the reduction of variability in expected delays explained by the workload relationship (29).
Rate Variability

![Graph](image)

- **Axes:**
  - Y-axis: $\mu/\Lambda$ (Rate Variability)
  - X-axis: Time of Day (Minutes)

- **Lines:**
  - Green line: MSFT
  - Purple line: ORCL
Conclusion

We propose a tractable model that incorporates order routing capability and flow heterogeneity with the high-frequency dynamics of LOBs

- characterize equilibrium
- establish state space collapse — fragmented market is coupled through workload
- provide supporting empirical evidence

Future directions:

- modeling cancellations
- modeling state-dependent adverse selection
- modeling two-sided markets
- welfare analysis