Realized Wavelet Jump-GARCH model: Can wavelet decomposition of volatility improve its forecasting?

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Motivation

- Spectral estimation of integrated volatility.
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- Use the wavelet theory for jump detection.
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- Use the wavelet theory for jump detection.
- But their work is quite restricted ($2^n$ sample length determines the bias of final estimator).
Realized volatility measurement in time-frequency space

BP futures volatility in time-frequency domain:
Scale: 5-10 min
Realized volatility measurement in time-frequency space

BP futures volatility in time-frequency domain:
Scale: 5-10 min, 10-20 min
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BP futures volatility in time-frequency domain:
Scale: 5-10 min, 10-20 min, 20-40 min
Realized volatility measurement in time-frequency space

BP futures volatility in time-frequency domain:
Scale: 5-10 min, 10-20 min, 20-40 min, 40-80 min
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BP futures volatility in time-frequency domain:
Scale: 5-10 min, 10-20 min, 20-40 min, 40-80 min, up to 1 day volatility
Realized volatility measurement in time-frequency space

BP futures volatility in time-frequency domain:
Scale: 5-10 min, 10-20 min, 20-40 min, 40-80 min, up to 1 day volatility and Jump variation
Realized volatility measurement in time-frequency space

BP futures volatility in time-frequency domain:
Total volatility (sum of all)
Motivation cont.

Theory for estimator available

Theory for estimator available


- Are these components valuable for forecasting?
Motivation cont.

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- We propose a forecasting model based on volatility decomposition to several investment horizons and jumps.
Motivation cont.

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- We use our time-frequency estimator robust to noise and jumps.
Motivation cont.

Theory for estimator available


- Are these components valuable for forecasting?
- We propose a forecasting model based on volatility decomposition to several investment horizons and jumps.
- We use our time-frequency estimator robust to noise and jumps.
- Propose Realized Jump-GARCH and Realized Wavelet Jump-GARCH models.
Why wavelets?

- Allowing for decomposition of the process to time-frequency space.
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- Allowing for jump detection.
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- Wavelet theory may be embedded into the stochastic processes.
- Antoniou and Gustafson (1999) compare wavelets with martingales as well as stochastic processes.
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- Allowing for jump detection.
- Wavelet theory may be embedded into the stochastic processes.
- Antoniou and Gustafson (1999) compare wavelets with martingales as well as stochastic processes.
- We utilize these results and bring them into the theory of quadratic variation.
Continuous wavelet transform

Definition

A function \( f \in L^2(\mathbb{R}) \) can be represented by the functions \( Wf \) such that,

\[
(Wf)(j, k) = \langle \psi_{j,k}, f \rangle = \left| j \right|^{-1/2} \int_{\mathbb{R}} \psi \left( \frac{s - k}{j} \right) f(s) ds
\]

(1)

where \( \langle ., . \rangle \) defines the \( L^2 \)-inner product and \( \psi_{j,k}(s) \) represents an orthogonal wavelet function \( \psi_{j,k}(s) = \left| j \right|^{-1/2} \psi\left( \frac{s-k}{j} \right) \) with a compact support with scale \( j \) and translation \( k \), where \( j \in \mathbb{R}^+, k \in \mathbb{R} \) and \( a \neq 0 \). Function \( W \) is called the continuous wavelet transform.
Martingale representation theorem

**Theorem 1**

For any univariate, square-integrable, continuous sample path, logarithmic prices process \((X_t)_{t\in[0,T]}\), which is not locally risk-less, there exists a representation for \(0 \leq t \leq T\):

\[
    r_t = \mu_t + M_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s
\]

where \(\mu_s\) is an integrable, predictable, and finite-variation stochastic process, \(\sigma_s\) is a strictly positive càdlàg stochastic process. \(\psi_{j,k} \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})\) represents the Daubechies (D4) wavelet function with a compact support.
Martingale representation theorem by wavelets

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\[
\begin{align*}
  r_t &= \mu_t + M_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s \\
  &= \int_0^t \int_0^\infty \int_\mathbb{R} \psi_{j,k}(s) \langle \psi_{j,k}, \mu_s \rangle dk \frac{1}{j^2} dj ds \\
  &\quad + \int_0^t \int_0^\infty \int_\mathbb{R} \psi_{j,k}(s) \langle \psi_{j,k}, \sigma_s \rangle dk \frac{1}{j^2} dj dW_s,
\end{align*}
\]

(2)

where \(\mu_s\) is an integrable, predictable, and finite-variation stochastic process, \(\sigma_s\) is a strictly positive càdlàg stochastic process.

\(\psi_{j,k} \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})\) represents the Daubechies (D4) wavelet function with a compact support.

**Proof** based on Calderón reconstruction formula.
Assumptions

- We work in a filtered probability space \((\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \in [0,T]}, \mathbb{P})\) satisfying usual conditions (i.e. Protter, 2004).

\[ X_t = \log S_t, \] is a true process underlying the stock market price \(S_t\) evolving continuously in time.

\[ (Y_t)_{t \in [0,T]} \] is an observed high-frequency data which contains microstructure noise \(\epsilon_t\).

\[ Y_t = X_t + \epsilon_t, \quad (3) \]

where \(Y_t\) is observed at times \(t_i = i/n, i, \ldots, n\), and \(\epsilon_t\) is zero mean i.i.d. noise with variance \(\eta^2\).

\[ X_t = \int_0^t \mu_s \, ds + \int_0^t \sigma_s \, dW_s + N_t \sum_{l=1}^J J_l, \quad (4) \]

\(J_t\) is a jump process usually assumed to be non-explosive Poisson process with \(N_t\) representing number of jumps in \(X\) and \(J_l\) jump size.
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Quadratic variation

Quadratic variation of the true log-price \((X_t)_{t \in [t-h,t]}\) composes of two parts: integrated volatility and jump variation.

\[
[X,X]_T = \int_{t-h}^{t} \sigma_s^2 ds + \sum_{t-h \leq s \leq t} J_s^2
\]

\(IV_{t,h}\)

Jump Var. (5)
Quadratic variation (wavelet-based)

Quadratic variation of the true log-price \((X_t)_{t \in [t-h,t]}\) composes of two parts: integrated volatility and jump variation.

\[
[X, X]_T = IV_{t,h} + \sum_{t-h \leq s \leq t} J_s^2
\]  

\[
= \int_{t-h}^t \sigma_s^2 ds + \sum_{t-h \leq s \leq t} J_s^2
\]

\[
= \int_{t-h}^t \int_0^\infty \int_\mathbb{R} \psi_{j,k}(s) \langle \psi_{j,k}, \sigma_s^2 \rangle dk \frac{1}{j^2} dj ds + \sum_{t-h \leq s \leq t} J_s^2
\]
Problem:
How to estimate the $IV_t,h$: $\int_{t-h}^{t} \sigma_s^2 ds$ from the data?
Estimation of true IV part

Problem:
How to estimate the $IV_{t,h}: \int_{t-h}^{t} \sigma^2_s ds$ from the data?
We measure $(Y_t)_{t \in [0,T]}$ which contains noise as well as jumps.
Wavelet based estimator of IV

We extend the results of Fan & Wang (2008) and Zhang, Mykland and Aït-Sahalia (2005) approach

- *Maximal overlap discrete wavelet transformation (MODWT)* and Daubechies (D4) filter is used instead of simple Haar.
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- $IV_{t,h}$ then is estimated in 2 steps:
  - Estimate jumps variation using wavelets.
  - Estimate IV using wavelet-based estimator on the basis of TSRV.
Jump Variation

Jump estimation

Let $\widetilde{W}_{1,i}$ be a 1st level wavelet coefficients of $[Y]_T$. If for some $\widetilde{W}_{1,i}$

$$|\widetilde{W}_{1,i}| > \frac{\text{median}\{|\widetilde{W}_{1,i}|, i = 1, \ldots, n\}}{0.6745} \sqrt{2 \log n}, \quad (6)$$

$\hat{\tau}_l = \{i\}$ is estimated jump location with size of $\bar{Y}_{\tau_{l}^+} - \bar{Y}_{\tau_{l}^-}$, the averages over $[\hat{\tau}_l, \hat{\tau}_l + \delta_n]$ and $[\hat{\tau}_l, \hat{\tau}_l - \delta_n]$ respectively, with $\delta_n > 0$ being a small neighborhood of estimated jump location $\hat{\tau}_l \pm \delta_n$
Jump Variation

Jump estimation

Let $\tilde{W}_{1,i}$ be a $1^{st}$ level wavelet coefficients of $[Y]_T$. If for some $\tilde{W}_{1,i}$

$$|\tilde{W}_{1,i}| > \frac{median\{|\tilde{W}_{1,i}|, i = 1, \ldots, n\}}{0.6745} \sqrt{2 \log n}, \quad (6)$$

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Jump variation

Jump variation $WJV_T = \sum_{l=1}^{N_t} J_l^2$ is then estimated by:

$$\overline{WJV}_T = \sum_{l=1}^{N_t} (\bar{Y}_{\hat{\tau}_l^+} - \bar{Y}_{\hat{\tau}_l^-})^2$$

(0.6745 is a robust estimate of standard deviation)
Jump Variation cont.

Consistency of Wavelet Jump estimator

$\widehat{WJV}_T$ estimator provides consistent measure of jumps over period of $[0, T]$

$$\text{plim}_{n \to \infty} \widehat{WJV}_T = \sum_{l=1}^{N_t} J_l^2$$

with the convergence rate $N^{-1/4}$.
Jump Variation cont.

Consistency of Wavelet Jump estimator

\( \hat{W}J^V_T \) estimator provides consistent measure of jumps over period of \([0, T]\)

\[
\text{plim}_{n \to \infty} \hat{W}J^V_T = \sum_{l=1}^{N_t} J_l^2
\]

with the convergence rate \(N^{-1/4}\).

- Knowing we are able to estimate jumps consistently, we utilize Zhang, Mykland and Aït-Sahalia (2005) TSRV on wavelet coefficients to deal with noise.
Wavelet based estimator of IV cont.

- The following estimator based on the MODWT of the jump-adjusted returns data $Y^{(J)}$ solves both problems.
Wavelet based estimator of IV cont.

- The following estimator based on the MODWT of the jump-adjusted returns data \( Y^{(J)} \) solves both problems.

**J-WTSRV estimator**

\[
\langle X, X \rangle_{T}^{(J-WTSRV)} = \left[ Y^{(J)}, Y^{(J)} \right]_{T}^{(WRV)} - \frac{\tilde{n}}{n} \left[ Y^{(J)}, Y^{(J)} \right]_{T}^{(all)},
\]

where \( \left[ Y^{(J)}, Y^{(J)} \right]_{T}^{(WRV)} = \frac{1}{K} \sum_{k=1}^{K} \sum_{j=s+1}^{J} \sum_{i=1}^{n} \tilde{W}_{j,i}^{2} \) obtained from MODWT wavelet coefficient estimates on the grid of size \( \tilde{n} = n/K \), \( n \) is number of intraday observations and \( J_s \) is number of scales we consider.
The following estimator based on the MODWT of the jump-adjusted returns data $Y^{(J)}$ solves both problems.

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\langle X, X \rangle_{T}^{(J-WTSRV)} = \left[ Y^{(J)}, Y^{(J)} \right]_{T}^{(WRV)} - \frac{\tilde{n}}{n} \left[ Y^{(J)}, Y^{(J)} \right]_{T}^{(all)},
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where $\left[ Y^{(J)}, Y^{(J)} \right]_{T}^{(WRV)} = \frac{1}{K} \sum_{k=1}^{K} \sum_{j=1}^{J_{s}+1} \sum_{i=1}^{n} \tilde{W}_{j,i}^{2}$ obtained from MODWT wavelet coefficient estimates on the grid of size $\tilde{n} = n/K$, $n$ is number of intraday observations and $J_{s}$ is number of scales we consider.

- J-WTSRV is unbiased and consistent estimator of $IV_{t,h}$
So we obtain a *true* volatility decomposed to several investment horizons and jumps.
Realized GARCH framework for forecasting

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- In the next step, we utilize it to build a forecasting model.
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- Several approaches in literature, i.e. Andersen, Bollerslev and Diebold (2007), Corsi, Pirino and Ren (2002), Corsi (2009).
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They improve GARCH by connecting latent volatility with realized measures.

Model introduces a measurement equation which ties the realized measure to latent volatility.
Realized GARCH framework for forecasting cont.

Hansen et al. (2011) model

\[ r_t = \sqrt{h_t} z_t, \quad (9) \]
\[ \log(h_t) = \omega + \beta \log(h_{t-1}) + \gamma \log(x_{t-1}) \quad (10) \]
\[ \log(x_t) = \xi + \psi \log(h_t) + \tau_1 z_t + \tau_2 z_t^2 + u_t, \quad (11) \]

where \( r_t \) is the return, \( x_t \) a realized measure of volatility, \( z_t \sim i.i.d(0, 1) \) and \( u_t \sim i.i.d(0, \sigma_u^2) \) with \( z_t \) and \( u_t \) being mutually independent, \( h_t = \text{var}(r_t|\mathcal{F}_{t-1}) \) with \( \mathcal{F}_t = \sigma(r_t, x_t, r_{t-1}, x_{t-1}, \ldots) \) and \( \tau(z) = \tau_1 z_t + \tau_2 z_t^2 \) is called leverage function.
Realized GARCH framework for forecasting cont.

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- The key is last equation - measurement equation.
We include jump component $WJV$ as well as decomposed volatilities $x_{j,t}$

The model

\begin{align*}
  r_t &= \sqrt{h_t} z_t, \\
  \log(h_t) &= \omega + \beta \log(h_{t-1}) + \sum_{j=1}^{J} \gamma_j \log(x_{j,t-1}) + \\
  &\quad + \gamma_J \log(1 + WJV_{t-1}), \\
  \log(x_t) &= \xi + \psi \log(h_t) + \tau_1 z_t + \tau_2 z_t^2 + u_t,
\end{align*}

Parameters are estimated using QMLE.
Empirical results

- British pound (GBP), Swiss franc (CHF) and Euro (EUR) futures.
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Empirical results

- British pound (GBP), Swiss franc (CHF) and Euro (EUR) futures.
- First we study if we can improve Realized GARCH with precise volatility measures.
- RV, BV, TSRV, RK and JWTGSRV are used.
- Then we include jump \( \Rightarrow \) Realized Jump GARCH
- Finally study the effect of decomposed volatility to 510 minutes, 1020 minutes, 2040 minutes and 4080 minutes, and the rest (80 minutes up to 1 day)
Table 3: Results for the GBP futures: in-sample fits of GARCH(1,1), Realized GARCH(1,1) with RV, BV, RK, TSRV, JWTTSRV, Realized (Wavelet)Jump-GARCH denoted as RJ-G and RWJ-G and Realized Jump-GARCH on JWTTSRV\textsubscript{j} decompositions. Robust standard errors are reported in parentheses.

<table>
<thead>
<tr>
<th>In-sample</th>
<th>GARCH</th>
<th>Realized GARCH</th>
<th>Realized (W)J-GARCH</th>
<th>Realized Jump-GARCH on JWTTSRV\textsubscript{j}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RV</td>
<td>BV</td>
<td>RK</td>
<td>JWSRV</td>
</tr>
<tr>
<td>(\omega)</td>
<td>0.147</td>
<td>0.155</td>
<td>0.130</td>
<td>0.153</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.029)</td>
<td>(0.024)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.271</td>
<td>0.283</td>
<td>0.267</td>
<td>0.288</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.033)</td>
<td>(0.028)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.599</td>
<td>0.568</td>
<td>0.691</td>
<td>0.573</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.039)</td>
<td>(0.035)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>(\tau_j)</td>
<td>0.049</td>
<td>0.059</td>
<td>0.249</td>
<td>0.331</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.073)</td>
<td>(0.049)</td>
<td>(0.054)</td>
</tr>
<tr>
<td>(\tau_{W1})</td>
<td>0.006</td>
<td>0.012</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>(\tau_{W2})</td>
<td>0.006</td>
<td>0.007</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.003)</td>
<td>(0.033)</td>
<td></td>
</tr>
<tr>
<td>(\tau_{W3})</td>
<td>0.121</td>
<td>0.121</td>
<td>0.121</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.017)</td>
<td>(0.016)</td>
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<tr>
<td>(\tau_{W4})</td>
<td>0.121</td>
<td>0.121</td>
<td>0.121</td>
<td></td>
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<tr>
<td></td>
<td>(0.019)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td>(\tau_{W5})</td>
<td>0.121</td>
<td>0.121</td>
<td>0.121</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td>(\xi)</td>
<td>-0.542</td>
<td>-0.549</td>
<td>-0.626</td>
<td>-0.533</td>
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Empirical results cont.

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JWTTSRV improves Realized GARCH considerably (fit + forecasting)
Empirical results cont.

Table 3: Results for the GBP futures: in-sample fits of GARCH(1,1), Realized GARCH(1,1) with RV, BV, RK, TSRV, JWTSRV, Realized (Wavelet) Jump-GARCH denoted as RJ-G and RWJ-G and Realized Jump-GARCH on JWTSRV decompositions. Robust standard errors are reported in parentheses.

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<th>Realized (W)J-GARCH</th>
<th>Realized Jump-GARCH on JWTSRV&lt;sub&gt;j&lt;/sub&gt;</th>
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Jumps further improve the results (fit + forecasting)
Empirical results cont.

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<td>(0.016)</td>
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<tr>
<td>( \tau_5 )</td>
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<td></td>
<td>(0.017)</td>
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<tr>
<td>( \xi )</td>
<td>-0.542</td>
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<td>(0.056)</td>
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<td>( \psi )</td>
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<td>(0.105)</td>
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<td>( \sigma_0 )</td>
<td>0.313</td>
<td>0.308</td>
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<tr>
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<td>(0.080)</td>
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<tr>
<td>( \tau_1 )</td>
<td>-0.041</td>
<td>-0.046</td>
<td>-0.030</td>
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<td></td>
<td>(0.012)</td>
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<tr>
<td>( \tau_2 )</td>
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<td>(0.009)</td>
<td>(0.011)</td>
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<tr>
<td>( l(r,x) )</td>
<td>-1185.123</td>
<td>-1172.427</td>
<td>-1327.555</td>
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<td>( I(r) )</td>
<td>-1001.735</td>
<td>-992.267</td>
<td>-992.605</td>
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<td>out-of-sample</td>
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<tr>
<td>( \alpha_M )</td>
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<td>-0.325</td>
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<td>(0.083)</td>
<td>(0.039)</td>
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<tr>
<td>( \beta_M )</td>
<td>1.607</td>
<td>0.946</td>
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<td>(0.039)</td>
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<td>( R_{Mx} )</td>
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<tr>
<td>( HMSE )</td>
<td>2.877</td>
<td>1.674</td>
<td>1.758</td>
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<td></td>
<td>(0.488)</td>
<td>(0.452)</td>
<td>(0.471)</td>
</tr>
<tr>
<td>( QLIKE )</td>
<td>0.488</td>
<td>0.452</td>
<td>0.471</td>
</tr>
</tbody>
</table>

Addition of volatility decomposition improves forecasting only slightly

separate 1st, 2nd and 3rd component carries the most information
Forecasting results

- Model shows the same performance on several Forex futures.
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- Still it is interesting to note that “fast scales” up to 30 minutes have most impact on the forecasts.
Further results and future work

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Realized Wavelet Jump-GARCH(1,1) online at:

Thank you very much for your attention

Questions?