Quantification of the High Level of Endogeneity and of Structural Regime Shifts in Financial and Commodity Markets

Vladimir Filimonov
Chair of Entrepreneurial Risks, ETH Zurich

with Didier Sornette (ETH Zurich); David Bicchetti and Nicolas Maystre (UNCTAD)
The opinions expressed in this work, including designation and terminology, are those of the authors and are not to be taken as the official views of the UNCTAD Secretariat or its Member States.
Financialization of commodities

Increasing market share of commodity speculators

1998
- Physical Hedger: 77%
- Index Speculator: 7%
- Traditional Speculator: 16%

2008
- Physical Hedger: 31%
- Index Speculator: 41%
- Traditional Speculator: 28%

Source: CFTC figures charts by Mike Masters, Better Markets.

Source: Goldman Sachs, Bloomberg, CFTC Commitments of Traders CIT Supplement
Volume traded per transaction

Brent Crude Oil

WTI

E-Mini S&P 500 Futures

Data source: TRTH

Typical market makers’ reaction time

Algorithmic and High-Frequency Trading

Adoption of algorithmic execution by asset classes

HFT trade volume as % of total market

U.S. ~65%
Europe ~54%
Asia ~19%

Source: Aite group
How does introduction and adoption of algorithmic (including HFT) trading affect price discovery mechanisms?

Is it possible to quantify the interplay between exogeneity (external impact) and endogeneity (internal self-excitation) in price formation?

How efficient are financial and financialized commodity markets?
Two views on the price discovery mechanism

Efficient Markets (exogenous dynamics)

Prices are just reflecting news: the market fully and instantaneously absorbs the flow of information and faithfully reflects it in asset prices.

In particular, financial crashes are the signature of exogenous negative news of large impact.

“Reflexivity” of markets (endogenous dynamics)

Markets are subjected to internal feedback loops (e.g. created by collective behavior such as herding or informational cascades).

Prices do influence the fundamentals and this newly-influenced set of fundamentals then proceed to change expectations, thus influencing prices.
Sources of reflexivity in financial and financialized markets

- Behavioral mechanisms such as imitation and informational cascades leading to herding;
- Speculation, based on technical analysis, including algorithmic trading;
- Hedging strategies (also increase cross-excitation between markets);
- Pricing of “structured products” such as ETFs (also contribute to cross-excitation)
- Methods of optimal portfolio execution and order splitting;
- Margin/leverage trading and margin-calls;
- High frequency trading (HFT) as a subset of algorithmic trading;
- Stop-loss orders and etc.
“As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools. In the absence of clear guidance from existing analytical frameworks, policy-makers had to place particular reliance on our experience”.

Jean-Claude Trichet (2010)
The test subject: HF price dynamics
The model: Self-excited Hawkes process

Self-excited Hawkes process is the point process whose intensity $\lambda_i(t)$ is conditional on its history:

$$\lambda(t) = \mu + n \sum_{t_i < t} \varphi(t - t_i)$$

Applications of the Hawkes model:
- High-frequency price dynamics
- Order book construction
- Critical events and estimation of VaR
- Default times in a portfolio of companies
- Triggered seismicity (earthquakes)
- Sequence of genes in DNA
- Epileptic seizures of brain
- Crime and violence propagation
Crucial parameter of the branching process is the “branching ratio” \( (n) \), which is defined as an average number of “daughters” per one “mother”.

For \( n < 1 \) system is **subcritical** (stationary evolution)
For \( n = 1 \) system is **critical** (tipping point)
For \( n > 1 \) system is **supercritical** (with prob.>0 will explode to infinity)

In subcritical regime, the branching ratio \( (n) \) is equal to the fraction of *endogenously generated events* among the whole population.
Calibration of the model

- **Maximum Likelihood method**
  Estimation of the parameters can be performed by maximizing log-likelihood function, which is given by the expression:

  \[
  \log L(t_1, \ldots, t_N) = -\int_0^T \lambda(t|\mathcal{F}_t)dt + \sum_{i=1}^N \log \lambda(t_i|\mathcal{F}_{t_i})
  \]

- **Residual analysis**
  Under the null hypothesis that the data \(\{t_i\}\) was generated by the Hawkes process with given parameters, the following transformed point process \(\{\tilde{t}_i\}\) should be Poisson with unit intensity:

  \[
  \tilde{t}_i = \int_0^{t_i} \lambda(t|\mathcal{F}_t)dt
  \]
Calibration issues. Kernel

- **Exponential kernel**
  \[ \phi(t) = \frac{1}{\tau} e^{-t/\tau} \chi(t) \]

- **Power law kernels**
  (a) Omori-type kernel
  \[ \phi(t) = \frac{\theta c^\theta}{(t + c)^{1+\theta}} \chi(t) \]
  (b) Power law kernel with cut-off
  \[ \phi(t) = \frac{\theta c^\theta}{t^{1+\theta}} \chi(t - c) \]
  (c) Approximate power law kernel
  \[ \phi(t) = \frac{1}{Z} \left[ \sum_{i=0}^{M-1} \frac{1}{\xi_i^{1+\theta}} \exp \left(-\frac{t}{\xi_i}\right) - S \exp \left(-\frac{t}{\xi_{-1}}\right) \right] , \quad \xi_i = cm^i \]
Calibration issues. Kernel: sensitivity to outliers

Empirical quantiles of inter-quote durations in E-mini S&P 500 Futures Contracts within RTH

<table>
<thead>
<tr>
<th>Date from</th>
<th>Date to</th>
<th>$Q_{90}$</th>
<th>$Q_{95}$</th>
<th>$Q_{99}$</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-02-2002</td>
<td>01-04-2002</td>
<td>13.7</td>
<td>20.6</td>
<td>41.7</td>
<td>458.9</td>
</tr>
<tr>
<td>01-02-2006</td>
<td>01-04-2006</td>
<td>23.3</td>
<td>39.6</td>
<td>90.4</td>
<td>933.1</td>
</tr>
<tr>
<td>01-02-2009</td>
<td>01-04-2009</td>
<td>5.1</td>
<td>8.7</td>
<td>19.4</td>
<td>329.9</td>
</tr>
<tr>
<td>01-02-2011</td>
<td>01-04-2011</td>
<td>4.2</td>
<td>10.8</td>
<td>38.7</td>
<td>888.0</td>
</tr>
</tbody>
</table>

Data source: TRTH

Theoretical quantiles of inter-event durations for Hawkes process with exponential kernel and $\mu=1$ and $n=0.7$

<table>
<thead>
<tr>
<th>$\tau_0$</th>
<th>$Q_{90}$</th>
<th>$Q_{95}$</th>
<th>$Q_{99}$</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.7</td>
<td>1.1</td>
<td>2.2</td>
<td>5.4</td>
</tr>
<tr>
<td>0.1</td>
<td>1.0</td>
<td>1.6</td>
<td>3.2</td>
<td>7.2</td>
</tr>
<tr>
<td>0.01</td>
<td>1.1</td>
<td>1.9</td>
<td>3.5</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Sensitivity of the estimation of branching ratio ($n$) to “outliers” in inter-event durations

Calibration issues. Kernel: regularization

Sensitivity of the estimation of branching ratio \( (n) \) to the mis-specification of the power law kernel

- Hawkes model with approximate power law kernel being calibrated on the data generated with Omori-type kernel
- Hawkes model with Omori-type kernel being calibrated on the data generated with approximate power law kernel

\[ \text{Estimated branching ratio, } n \]

\[ \text{Branching ratio, } n \]

Calibration issues. Multiple extrema

Surface of the reduced cost-function used for calibration of the Hawkes model on the mid-price changes of E-mini S&P 500 Contracts in March 1 - April 30, 2001, using the data randomized within millisecond intervals (see paper for details).

\[ \mu = 0.3031 \]
\[ n = 0.0751 \]
\[ c = 0.00028 \]
\[ \theta = 2.4604 \]

\[ \mu = 0.0150 \]
\[ n = 1.1054 \]
\[ c = 2.8089 \]
\[ \theta = 0.1442 \]

Data source: TRTH

Calibration issues. RTH and overnight trading

Fraction of total daily volume (left) and total daily mid-quote price changes (right) that is observed outside of Regular Trading Hours (9:30 to 16:15 CDT) on E-mini S&P 500 Futures Contracts.

Data source: TRTH

Calibration issues. Resolution of timestamps (I)

Histograms of the time between consecutive FAST/FIX packages (left panels) and overhead for the data processing (right panels) for E-mini S&P 500 Futures Contracts over RTH.

Data source: TRTH

Calibration issues. Resolution of timestamps (II)

Illustration of the randomization procedure, when the resolution of timestamps is mis-specified.

Bias in estimation of the branching ratio \( n \) that results from improper assumptions on the duration \( \Delta \) of randomization intervals, when real inter-packet time is 1 second.

- exponential kernel \( (n=0.5) \)
- power law kernel \( (n=0.5) \)
- Poisson process \( (n=0) \)


Left panels present the raw data (black bars) and the average intensity over the period of September 1–October 30, 2007 (red line).

Right panels present the unconditional intensity after “detrending” using the average intensity.

Data source: TRTH

Bias of the estimation of the branching ratio ($n$) in case of regime switch in background intensity (concatenation of 2 independent samples with $\mu_1=1$ and $\mu_2$, $n=1$)

Bias of the estimation of the branching ratio ($n$) in case of regime switch in branching ratio intensity (concatenation of 2 independent samples with $n_1=0.5$ and $n_2$)

Calibration issues. Nonstationarity (II)

Dynamics of daily numbers of mid-quote price changes counted over RTH for the Front Month Contract of the E-mini S&P 500 Futures (time period of February 1 to April 1 in three different years)

Data source: TRTH

Methodology

We split the entire interval of the analysis (2005-2012) into 10 minutes intervals, rolling them with a step of 1 minute within the RTH.

In each of these windows we have calibrated the Hawkes model with the short-term exponential kernel:

$$\lambda_t(t) = \mu + \frac{n}{\tau} \sum_{t_i < t} \exp \left( \frac{t - t_i}{\tau} \right)$$

on the timestamps of mid-quote price changes.

Each calibration resulted in a single estimation of the branching ration \((n)\).

We have performed residual analysis and rejected “bad” fits (using KS-test).

Collecting all estimates for each month (~6000-7000 estimates) we have averaged them to construct the “reflexivity index” for the given month.
Mechanisms of self-reflexivity

- High-frequency trading
- Stop-loss orders
- Algorithmic trading
- Optimal execution
- Margin calls
- Imitation
- Long-term herding
Benchmark: Financial markets (E-mini S&P 500)

Trading activity
proxied by volume and number of mid-price changes

Dynamics of price and volatility

Rate of exogenous events
(triggered by idiosyncratic “news”)

Branching ratio that quantifies reflexivity of the system
(fraction of endogenous events in the system)

Data source: TRTH

Crude Oil: Brent and WTI

Brent Crude (ICE Europe)

WTI (NYMEX)

• Filimonov V., Bicchetti D., Maystre N., Sornette D. (2013) Journal of International Money and Finance
Soft commodities: Soybean, Corn and Wheat

Soybean (CBOT)  Corn (CBOT)  Wheat (CBOT)

Daily volatility  Daily closing price

Year: 2005 - 2012

Data source: TRTH

April 27, 2010:
Significant fall of most of US markets following the cut of the credit rating of Greece and Portugal

May 6, 2010 ("flash-crash"): The activity of high-frequency traders of the S&P 500 E-mini futures contracts led to a dramatic fall in other markets

Volume and Trading activity behave similar in both cases

Branching ratio ("reflexivity index") reveals fundamental difference between two shocks

References


