Causal Model of Price and Inventory on a Market with a Market Maker

Martin Šmíd, Miloš Kopa

ÚTIA AS ČR, Pod Vodárenskou věží4, Praha 8, 182 08 Czech Republic

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The Setting

- A (representative) market maker (MM) posts (log) quotes $a_t$ and $b_t$ at equidistant (w.l.o.g. unit) intervals.

\[
\text{Liquidity traders post market orders at rate } \kappa \text{ (on both sides) with i.i.d. sizes.}
\]

\[
\text{Averagely rational traders post market i.i.d. orders with intensity } \lambda (a_t - \pi_t), \lambda (\pi_t - b_t) \text{ respectively,}
\]

\[
\lambda (z) = \max(r (1 - z/D), 0),
\]

where $\pi_t$ is a random walk (perhaps the log-fair price).
The Setting

- A (representative) market maker (MM) posts (log) quotes $a_t$ and $b_t$ at equidistant (w.l.o.g. unit) intervals.
- Liquidity traders post market orders at rate $\kappa$ (on both sides) with i.i.d. sizes.
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\[
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\]

where $\pi$ is a random walk (perhaps the log-fair price).

**Formally:** Denoting $\Xi_t$ is all the known past information

\[
X_{t+1}|\Xi_t \sim \text{CompoundPoisson} (\kappa + \lambda(a_t - \pi_t), \mathcal{D}),
\]

\[
Y_{t+1}|X_{t+1}, \Xi_t, \sim \text{CompoundPoisson} (\kappa + \lambda(\pi_t - b_t), \mathcal{D}),
\]

where $X$ and $Y$ is the sold, purchased volume, respectively and $\mathcal{D}$ is the distribution of order sizes.
The fair price \((\pi_1, \ldots, \pi_t)\) is unknown to MM but observed via a proxy

\[ e_1, \ldots, e_t \]

\[ \mathbb{E}(e_t - \pi_t|\Xi_{t-1}, X_t, Y_t) \sim N(0, \nu_e) \]
The Setting (cont.)

- The fair price \((\pi_1, \ldots, \pi_t)\) is unknown to MM but observed via a proxy

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\[ \mathbb{E}(e_t - \pi_t | \Xi_{t-1}, X_t, Y_t) \sim N(0, \nu_e) \]

- Moreover, MM observes the number of stocks he holds, i.e. all the information he has at his disposal is

\[ \xi_t = (N_1, e_1, \ldots, N_t, e_t), \quad \Delta N_t = Y_t - X_t \]
The fair price \((\pi_1, \ldots, \pi_t)\) is unknown to MM but observed via a proxy
\[ e_1, \ldots, e_t \]
\[ \mathbb{E}(e_t - \pi_t \mid \Xi_{t-1}, X_t, Y_t) \sim N(0, v_e) \]
Moreover, MM observes the number of stocks he holds, i.e. all the information he has at his disposal is
\[ \xi_t = (N_1, e_1, \ldots, N_t, e_t), \quad \Delta N_t = Y_t - X_t \]
a\_t and b\_t is a function of \(\xi_t\) \((\xi_t\text{-measurable})\).
The Setting (cont.)

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  \[ \xi_t = (N_1, e_1, \ldots, N_t, e_t), \quad \Delta N_t = Y_t - X_t \]

- \(a_t\) and \(b_t\) is a function of \(\xi_t\) (\(\xi_t\)-measurable).

**Approximation:** If the intensity \(r\) is high enough and

\[ a_t - h_t + \sqrt{v_{h,t}} \ll D, \quad h_t - b_t + \sqrt{v_{h,t}} \ll D \]

where \(h_t = \mathbb{E}(\pi_t | \xi_t)\), \(v_{h,t} = \text{var}(\pi_t - h_t | \xi_t)\), then
The Setting (cont.)

- The fair price \((\pi_1, \ldots, \pi_t)\) is unknown to MM but observed via a proxy
  \[ e_1, \ldots, e_t \]
  
  \[ \mathbb{E}(e_t - \pi_t | \Xi_{t-1}, X_t, Y_t) \sim N(0, v_e) \]

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where \(h_t = \mathbb{E}(\pi_t | \xi_t), v_{h,t} = \text{var}(\pi_t - h_t | \xi_t)\), then

- we may approximate the Compound Poisson distributions by the Normal ones
- we may take \(\lambda\) as linear
"Filter"

Theorem

Given suitable initial conditions,

\[
\begin{bmatrix}
\Delta \pi_t \\
e_t - \pi_t \\
\Delta N_t \\
h_t - \pi_t \\
\end{bmatrix} \xi_{t-1}
\]

\[
\sim \mathcal{N}
\begin{pmatrix}
0 \\
0 \\
k \delta_{t-1} \\
0 \\
\end{pmatrix},
\begin{pmatrix}
\nu_{\Delta \pi} & 0 & 0 & 0 \\
0 & \nu_e & 0 & 0 \\
0 & 0 & \nu_{N,t} & -k \nu_{h,t-1} \\
0 & 0 & -k \nu_{h,t-1} & \nu_{h,t-1} \\
\end{pmatrix}
\]

\[
\nu_{N,t} = \nu_N(\nu_{h,t-1}, \sigma_{t-1}) = 2s(\kappa + r - \frac{r \sigma_{t-1}}{D}) + k^2 \nu_{h,t-1}
\]

\[
k = 2 \frac{\mu r}{D}, \quad \delta_{\tau} = \frac{a_{\tau} + b_{\tau}}{2} - h_{\tau}, \quad \sigma_{\tau} = \frac{a_{\tau} - b_{\tau}}{2}
\]
Dynamics of $h$

**Theorem**

*Given suitable initial conditions*

$$\Delta h_t = -c_N(k^{-1} \Delta N_t - \delta_{t-1}) + c_e(e_t - h_{t-1})$$

*where*

$$c_N = c_N(v_{h,t-1}, \sigma_{t-1}) = \frac{v_{h,t-1}v_e}{u_t},$$

$$c_e = c_e(v_{h,t-1}, \sigma_{t-1}) = \frac{v_{h,t-1}k^{-2}v_{N,t} + v_{\Delta \pi}k^{-2}v_{N,t} + v_{h,t-1}v_{\Delta \pi}}{u_t}$$

$$u_t = v_{h,t-1}v_{\Delta \pi} + v_{h,t-1}k^{-2}v_{N,t} + v_{\Delta \pi}k^{-2}v_{N,t} + v_{h,t-1}v_e + k^{-2}v_{N,t}v_e$$
Maximization of discounted consumption while keeping the probability of bankruptcy small:
The Market Maker’s Decision Problem

Maximization of discounted consumption while keeping the probability of bankruptcy small:

\[ V_t(\xi_t) = \max_{a_\tau, b_\tau, C_\tau, \tau \geq t} \sum_{\tau = t}^{\infty} e^{-\rho(\tau-t)} \mathbb{E}(C_\tau | \xi_t) \]

\[ b_\tau \leq a_\tau, \quad t \leq \tau \quad \mathbb{P}[M_{\tau+1} < 0|\xi_\tau] \leq \gamma, \quad \mathbb{P}[N_{\tau+1} < 0|\xi_\tau] \leq \gamma. \]

\((a_\tau, b_\tau, C_\tau)\) is \(\sigma(\xi_\tau)\) measurable, \(t \leq \tau\).
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\[ (a_\tau, b_\tau, C_\tau) \text{ is } \sigma(\xi_\tau) \text{ measurable,} \quad t \leq \tau \]

where

- \( \rho \) is a discount factor
- \( \gamma \) is a prescribed level
- \( M_\tau \) is the amount of money at the time \( \tau \) fulfilling

\[ \Delta M_{\tau+1} = e^{a_\tau} X_{\tau+1} - e^{b_\tau} Y_{\tau+1} - C_\tau \]

- (recall that \( N_\tau \) is the amount of stocks at the time \( \tau \))
Theorem

Let $a_\tau, b_\tau, t \leq \tau$, are the optimal solutions of the problem. Then $a_\tau$ and $b_\tau$, generally being functions of $\xi_\tau$, are in fact functions of $N_\tau, v_{h,\tau}$ and $h_t$ only. In particular

\begin{align*}
a_\tau &= a(h_\tau, N_\tau, v_{h,\tau}) = h_\tau + \delta(N_\tau, v_{h,\tau}) + \sigma(N_\tau, v_{h,\tau}) \\
b_\tau &= b(h_\tau, N_\tau, v_{h,\tau}) = h_\tau + \delta(N_\tau, v_{h,\tau}) - \sigma(N_\tau, v_{h,\tau})
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\end{align*}
\]

"Taylor expansion" 

\[
\delta(N, v) \doteq d_0 + d_1 N + d_2 v
\]
Price decomposition

Idea from market microstructure: the transaction costs decompose into

- handling costs
- inventory costs
- adverse selection costs (i.e. costs of the uncertainty)
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Idea from market microstructure: the transaction costs decompose into:
- handling costs
- inventory costs
- adverse selection costs (i.e. costs of the uncertainty)

Analogously, the (log) midpoint price

\[ P_t = \frac{a_t + b_t}{2} = h_t - \delta(N_t, v_{h_{t-1}}) \]

can be written as

\[ P_T \doteq \pi_T + \eta_T + d_2 v_{\eta_T} + d_0 + d_1 N_T \]

\[ \eta_t = h_t - \pi_t \]
Price decomposition (cont.)

Decomposition of $\Delta P_t$

\[ \Delta P_t = \Delta \pi_t + d_1 \Delta N_t + c_{e,t}(\gamma_t - \eta_{t-1}) + c_{N,t}(k^{-1} \Delta N_{\tau} - \delta_{\tau-1}) + d_2 \Delta v_{\eta,\tau} \]

- $\Delta \pi_t$: fair price
- $\Delta N_t$: inventory
- $c_{e,t}(\gamma_t - \eta_{t-1})$: uncertainty
- $c_{N,t}(k^{-1} \Delta N_{\tau} - \delta_{\tau-1})$: uncertainty
- $d_2 \Delta v_{\eta,\tau}$: uncertainty
Price decomposition (cont.)

Decomposition of $\Delta P_t$

$$\Delta P_t = \underbrace{\Delta \pi_t}_{\text{fair price}} + \underbrace{d_1 \Delta N_t}_{\text{inventory}} + c_{e,t}(\gamma_t - \eta_{t-1}) + c_{N,t}(k^{-1} \Delta N_{\tau} - \delta_{\tau-1}) + d_2 \Delta v_{\eta,\tau}$$

Decomposition of volatility

$$\text{var}(\Delta P_t | \xi_{\tau-1}) = \underbrace{v_{\Delta \pi}}_{\text{fair price}} + \underbrace{d_1^2 v^*_{N,t}}_{\text{inventory}} + (2d_1c_{N,t}k^{-1} + c_{N,t}^2k^{-2})v^*_{N,t}d_1^2 + c_{e,t}v_\gamma$$

$$+ [(kd_1 + c_{N,t})^2 + c_{e,t}(kc_{e,t} + c_{N,t})]v_{h,t-1}$$

$v^*_{N,t} = 2s(\kappa + r - \frac{r\sigma_{t-1}}{D})$ is the variance of market orders if $\pi$ is known.
Assume the proxy $e$ is partially public:

$$e_t = \alpha e_t^p + (1 - \alpha) e_t^h, \quad \alpha \in [0, 1],$$

where $e_t^p$ and $e_t^v$ is, is not, respectively, available to the econometrician.
Econometrics of the Model

Assume the proxy $e$ is partially public:

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In addition, the econometrician observes

$$a_1, b_1, N_1, a_2, b_2, N_2, \ldots$$
Econometrics of the Model

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Goals:

- Validate the model
- Estimate the influences of inventory, and uncertainty to the price
- Decompose the volatility
Econometrics

After some algebra and approximation

\[ v_{h,t}(\sigma_t, v_{h,t-1}) \approx \omega_0 + \omega_1 \sigma_{t-1} + \omega_2 v_{h,t-1} \approx \frac{\omega_0}{1 - \omega_2} + \omega_1 \sum_{j=0}^{j_0} \omega^j_2 \sigma_{t-1-j} \]

we get

**Regressions**

\[ \Delta N_t = \phi_0 + \phi_N N_{t-1} + \phi_\sigma \sum_{i=2}^{j_0+2} \omega^{i-2}_2 \sigma_{t-i} + \mathcal{E}_t \]

\[ \Delta \Delta P_t = \psi_{\Delta N,0} \Delta N_t + \psi_{\Delta N,1} \Delta N_{t-1} + \psi_\sigma \Delta (e^p_t - P_{t-1}) \]

\[ + \psi_{1,\Delta \sigma} \Delta \sigma_1 + \psi_{\Delta \sigma} \sum_{i=2}^{j_0+1} \omega^{i-2}_2 \Delta \sigma_{t-i} + \mathcal{F}_t \]

where \( \mathcal{E}_t, \mathcal{F}_t \) are (inhomogeneous) MA processes, \( \phi \)’s and \( \psi \)’s are functions the model’s parameters.
The system may be estimated by two-stage OLS applied to sample with every second observation omitted.
Estimation (cont.)

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The parameters of interest may be retrieved as

\[ d_1 = k^{-1} \phi_N, \quad d_2 \omega_1 = \psi_{1, \Delta \sigma} \]
Estimation (cont.)

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- \( \psi_{\Delta\sigma} = d_2 \omega_1 \) i.e. \( \psi_{\Delta\sigma} \) indicates relation of inventory and spread
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- \( \psi_{\Delta\sigma} = d_2 \omega_1 \) i.e. \( \psi_{\Delta\sigma} \) indicates relation of inventory and spread
- \( d_2 \) and \( \omega_1 \) are not identified in our model.
Data

- One month (3/2009) of 10s trade and quote data by TickData.
Data

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- Three stocks:
  - GE  General Electric
  - MSFT  Microsoft
  - XOM  Exxon Mobile

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(Note that $N = Y - X$ is NOT traded volume $Q = X + Y$)
## XOM at ISE

<table>
<thead>
<tr>
<th>Volume/s: 5.79281</th>
<th>Trades/s: 0.05105</th>
<th>Avg. spread: 0.04800</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_0)</td>
<td>14.65459(3.34855)***</td>
<td></td>
</tr>
<tr>
<td>(\phi_N)</td>
<td>0.00001(0.00001)</td>
<td></td>
</tr>
<tr>
<td>(\phi_\sigma)</td>
<td>2746.40730(3469.53920)</td>
<td></td>
</tr>
<tr>
<td>(\phi_\sigma \omega_2)</td>
<td>3373.88480(3466.91130)</td>
<td></td>
</tr>
</tbody>
</table>

| \(\psi_\Delta N_0\) | 0.00293(0.01000)** |          |
| \(\psi_\Delta N_1\) | 0.00035(0.00093)   |          |
| \(\psi_p\)          | 0.02398(0.00392)***|          |
| \(\psi_\Delta \sigma\) | 0.06751(0.00489)***|          |
| \(\psi_\Delta \sigma \omega_2\) | 0.05368(0.00529)***|          |
| \(\psi_\Delta \sigma \omega_2^2\) | 0.02998(0.00487)***|          |

## XOM at NASDAQ OMX BX

<table>
<thead>
<tr>
<th>Volume/s: 3.95841</th>
<th>Trades/s: 0.05480</th>
<th>Avg. spread: 0.01974</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_0)</td>
<td>2.96881(0.96306)***</td>
<td></td>
</tr>
<tr>
<td>(\phi_N)</td>
<td>0.00002(0.00001)***</td>
<td></td>
</tr>
<tr>
<td>(\phi_\sigma)</td>
<td>8128.39450(2843.48450)**</td>
<td></td>
</tr>
<tr>
<td>(\phi_\sigma \omega_2)</td>
<td>4976.46180(2822.03400)*</td>
<td></td>
</tr>
</tbody>
</table>

| \(\psi_\Delta N_0\) | 0.00591(0.00261)* |          |
| \(\psi_\Delta N_1\) | 0.02115(0.00267)***|          |
| \(\psi_p\)          | 0.25824(0.00360)***|          |
| \(\psi_\Delta \sigma\) | 0.10427(0.01168)***|          |
| \(\psi_\Delta \sigma \omega_2\) | 0.04764(0.01234)***|          |
| \(\psi_\Delta \sigma \omega_2^2\) | 0.03385(0.01172)***|          |

## XOM at NASD ADF

<table>
<thead>
<tr>
<th>Volume/s: 25.81465</th>
<th>Trades/s: 0.15398</th>
<th>Avg. spread: 0.23069</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_0)</td>
<td>256.57184(6.27659)***</td>
<td></td>
</tr>
<tr>
<td>(\phi_N)</td>
<td>0.00001(0.00000)***</td>
<td></td>
</tr>
<tr>
<td>(\phi_\sigma)</td>
<td>5474.99130(198.45842)***</td>
<td></td>
</tr>
<tr>
<td>(\phi_\sigma \omega_2)</td>
<td>847.80466(922.16570)</td>
<td></td>
</tr>
</tbody>
</table>

| \(\psi_\Delta N_0\) | 0.01557(0.00270)*** |          |
| \(\psi_\Delta N_1\) | 0.00252(0.00273)***|          |
| \(\psi_p\)          | 0.23332(0.00784)*** |          |

## XOM at NYSE

<table>
<thead>
<tr>
<th>Volume/s: 84.10351</th>
<th>Trades/s: 0.55785</th>
<th>Avg. spread: 0.02380</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_0)</td>
<td>698.39672(39.13537)***</td>
<td></td>
</tr>
<tr>
<td>(\phi_N)</td>
<td>0.00000(0.00000)***</td>
<td></td>
</tr>
<tr>
<td>(\phi_\sigma)</td>
<td>404677.74000(80996.99700)***</td>
<td></td>
</tr>
<tr>
<td>(\phi_\sigma \omega_2)</td>
<td>97589.56600(81008.23400)</td>
<td></td>
</tr>
</tbody>
</table>

| \(\psi_\Delta N_0\) | 0.00036(0.00008)*** |          |
| \(\psi_\Delta N_1\) | 0.00035(0.00008)***|          |
| \(\psi_p\)          | 0.04306(0.00248)*** |          |
### MSFT at ISE

<table>
<thead>
<tr>
<th>Volume/s:</th>
<th>11.60129</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trades/s:</td>
<td>0.05852</td>
</tr>
<tr>
<td>Avg. spread:</td>
<td>0.01000</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\phi_0 &= 2.72471(20.87350) \\
\phi_N &= 0.00000(0.00001) \\
\phi_\sigma &= 76943.19800(25285.75300)** \\
\phi_\sigma \omega_2 &= -32317.84600(24997.17700)
\end{align*}
\]

\[
\begin{align*}
\psi \Delta N,0 &= -0.00060(0.00037) \\
\psi \Delta N,1 &= 0.00272(0.00039)** \\
\psi_\varphi &= -0.21109(0.00307)** \\
\psi \Delta \sigma &= 0.12752(0.01261)** \\
\psi \Delta \sigma \omega_2 &= 0.04988(0.01381)** \\
\psi \Delta \sigma \omega_2^2 &= 0.01612(0.01271)
\end{align*}
\]

### MSFT at NASDAQ OMX BX

<table>
<thead>
<tr>
<th>Volume/s:</th>
<th>11.77780</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trades/s:</td>
<td>0.06204</td>
</tr>
<tr>
<td>Avg. spread:</td>
<td>0.01209</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\phi_0 &= 92.48818(9.10278)** \\
\phi_N &= -0.00003(0.00001)** \\
\phi_\sigma &= 9950.86790(9204.94400) \\
\phi_\sigma \omega_2 &= 20951.83300(9192.25570*)
\end{align*}
\]

\[
\begin{align*}
\psi \Delta N,0 &= -0.00076(0.00043)* \\
\psi \Delta N,1 &= 0.00133(0.00045)** \\
\psi_\varphi &= -0.10504(0.00323)** \\
\psi \Delta \sigma &= 0.28347(0.00549)** \\
\psi \Delta \sigma \omega_2 &= 0.14306(0.00600)** \\
\psi \Delta \sigma \omega_2^2 &= 0.07557(0.00528)**
\end{align*}
\]

### MSFT at NASD ADF

<table>
<thead>
<tr>
<th>Volume/s:</th>
<th>3.91194</th>
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</thead>
<tbody>
<tr>
<td>Trades/s:</td>
<td>0.02172</td>
</tr>
<tr>
<td>Avg. spread:</td>
<td>0.06471</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\phi_0 &= 28.91630(6.11898)** \\
\phi_N &= -0.00003(0.00001)* \\
\phi_\sigma &= -910.46266(954.57441) \\
\phi_\sigma \omega_2 &= 15.13309(959.79665)
\end{align*}
\]

\[
\begin{align*}
\psi \Delta N,0 &= -0.00381(0.00285) \\
\psi \Delta N,1 &= -0.01445(0.00411)** \\
\psi_\varphi &= -0.00586(0.00828)
\end{align*}
\]

### MSFT at Chicago

<table>
<thead>
<tr>
<th>Volume/s:</th>
<th>0.89886</th>
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</thead>
<tbody>
<tr>
<td>Trades/s:</td>
<td>0.00214</td>
</tr>
<tr>
<td>Avg. spread:</td>
<td>0.21162</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\phi_0 &= 6.74415(4.31141) \\
\phi_N &= -0.00011(0.00012) \\
\phi_\sigma &= 111.07702(815.77233) \\
\phi_\sigma \omega_2 &= 948.50255(795.05940)
\end{align*}
\]

\[
\begin{align*}
\psi \Delta N,0 &= 0.01729(0.00658)** \\
\psi \Delta N,1 &= -0.00871(0.00620) \\
\psi_\varphi &= 0.05054(0.00998)**
\end{align*}
\]
Causal Model of Price and Inventory on a Market with a Market Maker

Econometrics
Model
Dynamics
Behavior of the Market Maker
Price Decomposition
Econometrics
Estimation Procedure
Data Results
Conclusions

Miloš Kopa

<table>
<thead>
<tr>
<th>GE at ISE</th>
<th>GE at NASDAQ OMX BX</th>
<th>GE at Chicago</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volume/s:</strong> 41.52370</td>
<td><strong>Volume/s:</strong> 11.36131</td>
<td><strong>Volume/s:</strong> 3.30688</td>
</tr>
<tr>
<td><strong>Trades/s:</strong> 0.13993</td>
<td><strong>Trades/s:</strong> 0.05147</td>
<td><strong>Trades/s:</strong> 0.01017</td>
</tr>
<tr>
<td><strong>Avg. spread:</strong> 0.00992</td>
<td><strong>Avg. spread:</strong> 0.01139</td>
<td><strong>Avg. spread:</strong> 0.03028</td>
</tr>
</tbody>
</table>

| | | |
| **φ₀** | - | 252.56481(101.80402)** | **φ₀** | 34.14363(10.88885)*** |
| **φₙ** | - | 0.00000(0.00001) | **φₙ** | 0.00001(0.00001)* |
| **φσ** | 163756.12000(63845.79100)** | **φσ** | 1614.73770(4902.21680)* |
| **φσω₂** | 225449.65000(63996.05900)*** | **φσω₂** | 373.94420(5013.58460) |

| | | |
| **ΔN₀** | 0.00015(0.00019) | **ΔN₀** | 0.00033(0.00088) |
| **ΔN₁** | 0.00054(0.00017)*** | **ΔN₁** | 0.00355(0.00089)*** |
| **Δp** | 0.06646(0.00220)*** | **Δp** | 0.07213(0.00213)*** |
| **Δσ** | 0.05996(0.01760)*** | **Δσ** | 0.16826(0.00594)*** |
| **Δσω₂** | 0.00599(0.01978) | **Δσω₂** | 0.06071(0.00653)*** |
| **Δσω₂** | - 0.01592(0.01755) | **Δσω₂** | 0.02590(0.00589)*** |

| | | |
| **Volume/s:** 23.04329 | **Volume/s:** 3.30688 | **Volume/s:** 3.30688 |
| **Trades/s:** 0.09504 | **Trades/s:** 0.01017 | **Trades/s:** 0.01017 |
| **Avg. spread:** 0.02971 | **Avg. spread:** 0.03028 | **Avg. spread:** 0.03028 |

| | | |
| **φ₀** | 217.96378(17.61001)*** | **φ₀** | 11.63743(4.30832)** |
| **φₙ** | 0.00005(0.00001)*** | **φₙ** | 0.00003(0.00002) |
| **φσ** | 6564.37130(2404.18670)** | **φσ** | 23.99393(486.71636) |
| **φσω₂** | 627.39315(2360.57700) | **φσω₂** | 79.64745(486.72104) |

| | | |
| **ΔN₀** | - 0.00279(0.00144)* | **ΔN₀** | - 0.07437(0.15615) |
| **ΔN₁** | - 0.01034(0.00171)*** | **ΔN₁** | - 0.07224(0.18958) |
| **Δp** | - 0.08855(0.00665)*** | **Δp** | 0.09273(0.11443) |

Raw results (selected)
### Influences of inventory (\(N\)) and uncertainty (\(\sigma\))

<table>
<thead>
<tr>
<th></th>
<th>ISE</th>
<th>NAS</th>
<th>NSE</th>
<th>NAS</th>
<th>Chic</th>
<th>NY</th>
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<tbody>
<tr>
<td>(N)</td>
<td>↑</td>
<td>⬆</td>
<td>↓</td>
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<td>(\sigma)</td>
<td>↑</td>
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<table>
<thead>
<tr>
<th></th>
<th>XOM</th>
<th>MSFT</th>
<th>GE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N)</td>
<td>↑</td>
<td>⬇</td>
<td>⬇</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>↑</td>
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<td>⬆</td>
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</table>

The conjecture that "MM sell out extra inventory by lowering the prices" mostly confirmed. An influence of spread (perhaps also of uncertainty) mostly confirmed.

Martin Šmíd, Miloš Kopa

Causal Model of Price and Inventory on a Market with a Market Maker
Influences of inventory ($N$) and uncertainty ($\sigma$)

<table>
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<tr>
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<th>NAS Chicago</th>
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<th>NAS T</th>
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<tbody>
<tr>
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<td>O B</td>
<td>ADF</td>
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</table>

**XOM**

<table>
<thead>
<tr>
<th>$N$</th>
<th>↑</th>
<th>↑</th>
<th>↓ ***</th>
<th>↓ ***</th>
<th>↓ ***</th>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>↑***</td>
<td>↑***</td>
<td>↑***</td>
<td>↑***</td>
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**MSFT**

<table>
<thead>
<tr>
<th>$N$</th>
<th>↑</th>
<th>↓ ***</th>
<th>↓ **</th>
<th>↓ *</th>
<th>↓</th>
<th></th>
<th>↑</th>
<th></th>
<th>↓ ***</th>
</tr>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>↑***</td>
<td>↑***</td>
<td>↑***</td>
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**GE**

<table>
<thead>
<tr>
<th>$N$</th>
<th>↓</th>
<th>↓ *</th>
<th>↓ ***</th>
<th>↓ ***</th>
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<tr>
<td>$\sigma$</td>
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- The conjecture that "MM sell out extra inventory by lowering the prices" mostly confirmed"
Influences of inventory ($N$) and uncertainty ($\sigma$)

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<td></td>
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<td></td>
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</tr>
<tr>
<td>$N$</td>
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<td>↑</td>
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<td>***</td>
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<td>***</td>
<td>↑</td>
<td>***</td>
</tr>
<tr>
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<td>↑</td>
<td></td>
</tr>
</tbody>
</table>

**XOM**

**MSFT**

**GE**

- The conjecture that "MM sell out extra inventory by lowering the prices" mostly confirmed.
- An influence of spread (perhaps also of uncertainty) mostly confirmed.
Conclusions

Model with rational partially informed MM and averagely rational LT formulated.
Conclusions

Model with rational partially informed MM and averagely rational LT formulated.

- Dynamics of involved variables derived
Conclusions

Model with rational partially informed MM and averagely rational LT formulated.

- Dynamics of involved variables derived
- Price decomposition proposed (its concrete estimation being a future work)
Conclusions

Model with rational partially informed MM and averagely rational LT formulated.

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- Reverse inventory effects and effects of spread confirmed.
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Model with rational partially informed MM and averagely rational LT formulated.

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- Reverse inventory effects and effects of spread confirmed.

Thanks for attention!