The views represented herein are the author’s own views and do not necessarily represent the views of Morgan Stanley or its affiliates and are not a product of Morgan Stanley Research.
Delta Hedging and HFT

- Delta hedging is a well known trading strategy intended to reduce the risk inherent in selling an option.

- Delta hedging is a high frequency trading strategy. The standard theoretical treatment has the hedger trading in the option’s underlying asset continuously over time.

- Under some strong assumptions such as no frictions, no price jumps, and a predictable volatility process, all of the risk from selling an option can be removed by continuously trading in the option’s underlying asset.
A quote often misattributed to Casey Stengel is:

_In theory there is no difference between theory and practice; in practice there is._

The assumptions that lead to complete risk elimination via delta hedging are clearly false in reality. In practice, frictions exist, prices jump, no one can perfectly predict volatility and no one will ever be able to trade continuously.

The statistician George Box wrote:

"All models are wrong but some are useful".

In this talk, we address the question of whether the Black Merton Scholes model is useful.
The Mathematical Equation That Caused the Banks to Crash

- The Feb 11, 2012 issue of the Guardian, a respected British newspaper, has an article with the same title as this slide.
- The article was written by Ian Stewart, an Emeritus Professor of Mathematics at the University of Warwick, England, and a widely known popular-science and science-fiction writer.
- The article has a photo of the Black Scholes partial differential equation:

\[
\frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV = 0
\]

- The article is clear that this is the equation referred to in the title.
Here are two quotes from the article:

The equation itself wasn’t the real problem. It was useful, it was precise, and its limitations were clearly stated. It provided an industry-standard method to assess the likely value of a financial derivative. So derivatives could be traded before they matured. The formula was fine if you used it sensibly and abandoned it when market conditions weren’t appropriate. The trouble was its potential for abuse.

Was an equation to blame for the financial crash, then? Yes and no. Black-Scholes may have contributed to the crash, but only because it was abused. In any case, the equation was just one ingredient in a rich stew of financial irresponsibility, political ineptitude, perverse incentives and lax regulation.
On October 22, 2015, Bloomberg reporter Noah Smith published an opinion piece with the same title as this slide.

Here are two quotes from the article:

*If you really want to see a field that took a hit from 2008, look at financial engineering.*

*Why did financial engineering models fail so disastrously? Essentially, people had too much confidence in them. Whereas the private sector basically didn’t trust economists’ models at all, traders trusted financial engineers’ models so much that they were willing to bet that they would hold true down to razor thin precision. Since then, people have called for less leverage in the financial system. Leverage amplifies bets, so this is essentially just a call to hedge all bets based on financial engineering models. But how much to hedge? There’s no clear answer.*
Here are two more quotes from the recent Bloomberg View article:

What this really means is that everyone should simply put much less faith in financial engineering. Leverage should be reduced, and making bets on derivatives should simply become a much, much smaller part of what the financial industry does. So it wasn’t economics that took the biggest hit from the crisis. Financial engineering, once revered and relied upon by many of the smartest people on the planet, has been humbled.

This column does not necessarily reflect the opinion of the editorial board or Bloomberg LP and its owners.
The most well known model in financial engineering is the Black-Merton-Scholes (BMS) model.

In 1973, call options on individual stocks were listed on the Chicago Board Options Exchange. In the same year, seminal papers by Black and Scholes [1] and by Merton[2] were published.

Robert Merton and Myron Scholes won the Nobel prize in Economics for the model in 1997; Fischer Black had died in 1995.

This model says that prices evolve continuously over time, the volatility of the underlying is constant, and that a trader can completely remove the risk of writing a call by continuously rebalancing shareholdings to neutralize the call delta.
One of the most important inputs to the formula for delta is the volatility of the underlying asset.

Given a single option price at some initial date, the option’s implied volatility can be calculated numerically.

If the BMS model is absolutely correct, this implied volatility can be used to obtain deltas for all strikes and maturities on all subsequent dates.

No one believes the model to this degree. In practice, the volatility used to compute delta at a particular time, strike price, and maturity date is the implied volatility for just that time, strike price, and maturity date.

It is common to also attempt a correction which reflects the slope of implied volatility in strike at the money. In what follows, we do not attempt any skew correction.
We performed the following experiment using historical S&P 500 options data from Jan 1996 - Jan 2015:

- Sell a one-month at-the-money call option
- Record the P&L from three strategies:
  1. **Sell and Hold**: Just hold the option short.
  2. **Static Delta Hedge**: Perform a delta hedge initially, but do no re-hedging.
  3. **Daily Delta Hedge**: Restore the delta to zero at the end of each day.

We record the P&L each month, then calculate the variance of P&L for each of the 3 strategies over the 19 year period.

By what fraction do you think the variance of P&L is reduced as we switch from sell-and-hold to daily delta hedge?

- [A] No reduction
- [B] 0 to 1/3
- [C] 1/3 to 2/3
- [D] 2/3 to 100%
Unheded, the standard deviation of P&L can exceed the option price.

Un-rebalanced, the static delta hedge deteriorates over time, but still removes 70% of the variance over one month.

Rebalanced daily, 95% of the variance can be removed on average over our sample period.

The answer is “D.”
Dynamic delta hedging works equally well on both short and long-term call options.

To gauge the stability of this result over time, we calculate the variance reduction for one month P&L for different calendar years.

The reduction in variance deteriorates somewhat in times of crisis.
Dynamic delta hedging works equally well on both short and long-term puts.

The reduction in put option variance works well in the recent financial crisis, but there is some deterioration of the hedge’s performance in early 1997.
• Improvements are smaller if the options are too far out of the money and never get close to the money, but the variance is small even without hedging.
Hedging puts at different maturities and time periods

1. **ATM**

2. **95% OTM**

   - Improvements are smaller if the options are too far out of money.

Carr & Wu

October 30, 2015
Some popular press has blamed the crisis on the models used in financial engineering.

The large number of papers citing deficiencies in the BMS model could lead one to believe that this model caused the crisis.

Contrary to this view, the hedging effectiveness of this simple model has been consistently high before, during, and after the crisis.

Dynamic delta hedging works equally well on both short and long-term options.

It works well for both calls and puts and it works well for OTM options, too.

We conclude that the BMS model is both wrong and useful.


Disclaimer

The information herein has been prepared solely for informational purposes and is not an offer to buy or sell or a solicitation of an offer to buy or sell any security or instrument or to participate in any trading strategy. Any such offer would be made only after a prospective participant had completed its own independent investigation of the securities, instruments or transactions and received all information it required to make its own investment decision, including, where applicable, a review of any offering circular or memorandum describing such security or instrument, which would contain material information not contained herein and to which prospective participants are referred. No representation or warranty can be given with respect to the accuracy or completeness of the information herein, or that any future offer of securities, instruments or transactions will conform to the terms hereof. Morgan Stanley and its affiliates disclaim any and all liability relating to this information. Morgan Stanley, its affiliates and others associated with it may have positions in, and may effect transactions in, securities and instruments of issuers mentioned herein and may also perform or seek to perform investment banking services for the issuers of such securities and instruments.
The information herein may contain general, summary discussions of certain tax, regulatory, accounting and/or legal issues relevant to the proposed transaction. Any such discussion is necessarily generic and may not be applicable to, or complete for, any particular recipient’s specific facts and circumstances. Morgan Stanley is not offering and does not purport to offer tax, regulatory, accounting or legal advice and this information should not be relied upon as such. Prior to entering into any proposed transaction, recipients should determine, in consultation with their own legal, tax, regulatory and accounting advisors, the economic risks and merits, as well as the legal, tax, regulatory and accounting characteristics and consequences, of the transaction.
Notwithstanding any other express or implied agreement, arrangement, or understanding to the contrary, Morgan Stanley and each recipient hereof are deemed to agree that both Morgan Stanley and such recipient (and their respective employees, representatives, and other agents) may disclose to any and all persons, without limitation of any kind, the U.S. federal income tax treatment of the securities, instruments or transactions described herein and any fact relating to the structure of the securities, instruments or transactions that may be relevant to understanding such tax treatment, and all materials of any kind (including opinions or other tax analyses) that are provided to such person relating to such tax treatment and tax structure, except to the extent confidentiality is reasonably necessary to comply with securities laws (including, where applicable, confidentiality regarding the identity of an issuer of securities or its affiliates, agents and advisors).
Disclaimer (Con’d)

The projections or other estimates in these materials (if any), including estimates of returns or performance, are forward-looking statements based upon certain assumptions and are preliminary in nature. Any assumptions used in any such projection or estimate that were provided by a recipient are noted herein. Actual results are difficult to predict and may depend upon events outside the issuers or Morgan Stanley’s control. Actual events may differ from those assumed and changes to any assumptions may have a material impact on any projections or estimates. Other events not taken into account may occur and may significantly affect the analysis. Certain assumptions may have been made for modeling purposes only to simplify the presentation and/or calculation of any projections or estimates, and Morgan Stanley does not represent that any such assumptions will reflect actual future events. Accordingly, there can be no assurance that estimated returns or projections will be realized or that actual returns or performance results will not be materially different than those estimated herein. Any such estimated returns and projections should be viewed as hypothetical.

Recipients should conduct their own analysis, using such assumptions as they deem appropriate, and should fully consider other available information in making a decision regarding these securities, instruments or transactions. Past performance is not necessarily indicative of future results. Price and availability are subject to change without notice.
The offer or sale of securities, instruments or transactions may be restricted by law. Additionally, transfers of any such securities, instruments or transactions may be limited by law or the terms thereof. Unless specifically noted herein, neither Morgan Stanley nor any issuer of securities or instruments has taken or will take any action in any jurisdiction that would permit a public offering of securities or instruments, or possession or distribution of any offering material in relation thereto, in any country or jurisdiction where action for such purpose is required. Recipients are required to inform themselves of and comply with any legal or contractual restrictions on their purchase, holding, sale, exercise of rights or performance of obligations under any transaction. Morgan Stanley does not undertake or have any responsibility to notify you of any changes to the attached information. With respect to any recipient in the U.K., the information herein has been issued by Morgan Stanley & Co. International Limited, regulated by the U.K. Financial Services Authority. THIS COMMUNICATION IS DIRECTED IN THE UK TO THOSE PERSONS WHO ARE MARKET COUNTER PARTIES OR INTERMEDIATE CUSTOMERS (AS DEFINED IN THE UK FINANCIAL SERVICES AUTHORITYS RULES). ADDITIONAL INFORMATION IS AVAILABLE UPON REQUEST.