Realized Variance: Empirical Forex Analysis and Estimator Based on Linear Regression

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Objectives

1. To estimate the integrated variance of a financial asset in the presence of microstructure noise.
2. To investigate the structure of microstructure noise.
Efficient Price and Microstructure Noise

- We consider the efficient price to follow continuous Itô semimartingale [2]

\[ p_t^* = p_0^* + \int_0^t \mu(s) \, ds + \int_0^t \sigma(s) \, dW_s. \]

- Instead of \( p_t^* \) process we observe discrete price process

\[ p_{\Delta N n} = p_{\Delta N n}^* + \varepsilon_{\Delta N n} \quad \text{for} \quad n = 1, \ldots, N \quad \text{and} \quad \Delta N = \frac{1}{N - 1} \]

contaminated by the additive microstructure noise
\( \varepsilon_{\Delta N n} \sim (0, \omega^2) \) [2].

- The noise \( \varepsilon_{\Delta N n} \) can be correlated with efficient price \( p_t^* \).
- The noise \( \varepsilon_{\Delta N n} \) can be serially correlated.
Integrated Variance and Realized Variance

- The integrated variance \( (IV) \) is defined as
  \[
  IV(0, T) = \int_0^T \sigma^2(s) \, ds.
  \]

- Simple estimator of IV is the realized variance \( (RV) \)
  \[
  RV_N = \sum_{n=1}^{N} (p_{\Delta N n} - p_{\Delta N (n-1)})^2.
  \]
Bias of Realized Variance

- In the presence of microstructure noise the expected value of RV \([1]\) is

\[
f_N := \mathbb{E}[RV_N] = IV + 2N\left(2\rho(0) - 2\rho(\Delta N)\right) + N\left(2\pi(0) - 2\pi(\Delta N)\right).
\]

- Function \(\rho(t)\) is the covariance function between \(p_{\Delta NN}^\ast\) and \(\varepsilon_{\Delta NN-t}\). It can have correlation structures described for example in [3].
- Function \(\pi(t)\) is the autocovariance function of \(\varepsilon_{\Delta NN}\). In this poster we consider the independent white noise and Ornstein-Uhlenbeck noise.
- RV is significantly biased under any noise setting.
General Idea of Estimator

- First we compute $RV_N$ for different numbers of observations $N = N_1, \ldots, N_M$ (i.e. different values of $\Delta N$).
  - We select numbers of observations from a fixed grid with equidistant elements.
- Then we model dependency of $RV_N$ on $N$ using regression model

$$RV_N = f_N(IV, \omega^2, \ldots) + \chi_N \quad \text{for} \quad N = N_1, \ldots, N_M$$

with coefficients $IV$ (the estimation of integrated variance), $\omega^2$ (the estimation of microstructure noise variance) and other parameters specific to some $\rho(t)$ or $\pi(t)$ structures.
Regression Assumptions

- To avoid heteroscedasticity and autocorrelation we use generalized regression with variance matrix of residuals. We empirically estimate this matrix from historical data using residuals of regression models with unit matrices as variance matrices.

- In case of white noise, the $RV_N$ is asymptotically normal. To some degree this advocates the use of regression with normally distributed residuals.
Model Specifications

- For the white noise structure we model RV using linear regression
  \[ RV_N = IV + 2N\omega^2 + \chi_N \quad \text{for} \quad N = N_1, \ldots, N_M \]
  with coefficients \( IV \) and \( \omega^2 \).

- For the Ornstein-Uhlenbeck structure we use non-linear regression
  \[ RV_N = IV + 2N\omega^2 - 2N\omega^2 e^{-\frac{1}{\tau(N-1)}} + \chi_N \quad \text{for} \quad N = N_1, \ldots, N_M \]
  with coefficients \( IV \), \( \omega^2 \) and \( \tau \) (correlation time).
Signature Plot

- Forex pairs exhibit non-linear RV bias.
- In some cases we observe decreasing curve in RV signature plots indicating microstructure noise negatively correlated with efficient price [1].
Regression Estimations: Lower Frequencies


Fitted RV bias – white noise (blue solid line) and Ornstein–Uhlenbeck structure (red solid line)
IV estimates – white noise (blue dashed line) and Ornstein–Uhlenbeck structure (red dashed line)
Regression Estimations: Higher Frequencies


Fitted RV bias – WN (blue solid line), OU (red solid line) and correlated OU (green solid line)
IV estimates – WN (blue dashed line), OU (red dashed line) and correlated OU (green dashed line)
Results and Future Research

1. Proposed method allows both the estimation of IV and analysis of the structure of microstructure noise.

2. Estimating IV of Forex prices require modelling noise as serially correlated and for higher frequencies also as correlated with efficient price.

3. Further research will consist of extending the regression analysis by considering more noise structures, using proposed approach for more real cases and comparing proposed method to other IV estimators.


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Thank you for your attention.